

H^∞ Control of Fuzzy Impulsive Systems with Quantized Feedback

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Abstract—This paper is concerned with the problem of H^∞ control of fuzzy nonlinear impulsive systems with quantized feedback. New results on the H^∞ feedback control are established for one class of fuzzy nonlinear uncertain impulsive systems and one class of fuzzy nonlinear impulsive systems with nonlinear uncertainties by choosing appropriately quantized strategies and applying Lyapunov function approach, respectively.

Index Terms—quantized feedback; fuzzy impulsive systems; Lyapunov functions; H^∞ control.

I. INTRODUCTION

Considerable efforts have been devoted to the study of quantized systems in recent years, for instance, see the papers in Brockett and Liberzon (2000), Ishii and Francis (2002, 2003), Liberzon (2003), Liu and Elia (2004), Ishii and Basar (2005), and the references therein. Among these results, mainly two approaches for studying control problems with quantized feedback are chosen, which are called static quantization policies (e.g., Delchamps, 1990; Elia and Mitter, 2001; Fu and Xie, 2005; Claudio De Persis, 2005) and dynamic quantization policies (e.g., Brockett and Liberzon, 2000; Tatikonda and Mitter, 2004).

For the problem of H^∞ control systems with quantized feedback, Guisheng Zhai, Xinkai Chen, Joe Imae and Tomoaki Kobayashi (2006) deal with analysis and design of H^∞ feedback control systems with two quantized signals. Huijun Gao and Tongwen Chen (2008) present a new approach to quantized feedback control systems which, both single- and multiple-input cases considered, provide for stability and H^∞ performance analysis as well as controller synthesis for discrete-time state-feedback control systems with logarithmic quantizers. The most significant feature is the utilization of a quantization dependent Lyapunov function. Francesca Ceragioli and Claudio De Persis (2007) discuss discontinuous stabilization of nonlinear systems with quantized and switching controls, i.e. considering the classical problem of stabilizing nonlinear systems in the case the

control laws take values in a discrete set. H^∞ performance analysis of affine nonlinear systems is also given.

Considerable attention, however, have been paid toward the study of fuzzy systems, for instance, see the papers T.Takagi and M.Sugeno (1985), Tong R.M. (1977), Kazuo Tanaka, Takayuki Ikeda, and Hua O. Wang (1998) and the references therein. Among these results, a significant model is the T-S model proposed by T.Takagi and M.Sugeno (1985) which is described by fuzzy IF-THEN rules. From the point of view of system analysis, T-S model is appealing since the stability and performance characteristics of the system can be analyzed using a Lyapunov approach including the quadratic Lyapunov functions (e.g., E. Kim and H. Lee, 2000; Tanaka K, Ikeda T, and Hua O. Wang, 1998) and the nonquadratic ones (e.g., Thierry Marie Guerra and Laurent Vermeiren, 2004).

In this paper, we concentrate on the problem of H^∞ feedback control of two classes of T-S fuzzy impulsive systems via fuzzy quantized feedback. New results on the H^∞ feedback control of one class of fuzzy nonlinear uncertain impulsive systems and one class of fuzzy nonlinear impulsive systems with nonlinear uncertainties are presented by choosing appropriately quantized strategies and applying the Lyapunov function approach, respectively.

The paper is organized as follows. Section II gives the concept of quantizer, some notations and problem statement. Section III presents new results on H^∞ feedback control of one class of fuzzy impulsive systems with non-quantized feedback. New results on analysis and design of H^∞ control of one class of fuzzy nonlinear uncertain impulsive systems with quantized feedback are presented in Section IV. Section V deals with analysis and design of H^∞ control of one class of fuzzy nonlinear impulsive systems with nonlinear uncertainties with quantized feedback. Conclusions are presented in section VI.

II. PROBLEM STATEMENT

In this section, some notations and definition of quantizer are introduced and the problem statement is given.

Firstly, we give the definition of a quantizer with general form as in Liberzon, D. (2003), Ishii, H. and Francis, B.A. (2003). Let $z \in R^l$ be the variable being quantized. A quantizer is defined as a piecewise constant function $q: R^l \rightarrow D$, where D is a finite subset of $z \in R^l$. This leads to a partition of $z \in R^l$ into a finite number of quantization regions of the form $\{z \in R^l : q(z) = i\}$, $i \in D$. The quantization regions are not assumed to have any particular shapes. We assume that there exist positive real numbers M and Δ such that the following conditions hold:

$$|z| \leq M \Rightarrow |q(z) - z| \leq \Delta; \tag{1}$$

$$|z| > M \Rightarrow |q(z)| > M - \Delta. \tag{2}$$

Throughout this paper, we denote by $|\cdot|$ the standard Euclidean norm in the n -dimensional vector space R^n , and denote by $\|\cdot\|$ the corresponding induced matrix norm in $R^{n \times n}$. Condition (1) gives a bound on the quantization error when the quantizer does not saturate. Condition (2) provides a way to detect the possibility of saturation. We will refer to M and Δ as the range of q and the quantization error, respectively. We also assume that $\{x : q(x) = 0\}$ for x in some neighborhood of the origin which is needed to preserve the origin as an equilibrium.

In the control strategy to be developed below, we will use quantized measurements of the form as in Liberzon, D. (2003), Ishii, H. and Francis, B.A. (2003).

$$q_\mu(z) := \mu q\left(\frac{z}{\mu}\right) \tag{3}$$

where μ is an adjustable parameter, called the "zoom" variable, that is update at discrete instants of time.

To be convenient, we denote $\sum_{i,j=1}^r := \sum_{i=1}^r \sum_{j=1}^r$, $h_i := h_i(x(t))$, $w_i := w_i(x(t))$, $h_i^{q_\mu(x)} := h_i(q_\mu(x(t)))$ and $h_i^{q_\mu(y)} := h_i(q_\mu(y(t)))$.

The T-S fuzzy system, suggested by T.Takagi and M.Sugeno (1985), can represent a general class of nonlinear systems. It is based on "fuzzy partition" of input space and it can be viewed as the expansion of piecewise linear partition.

Consider a uncertain impulsive nonlinear dynamic multi-input-multi-output system modeled by the T-S fuzzy system, which can be represented by the following forms:

• IF-THEN form:

$$\begin{aligned} R_i : & \text{ IF } x_1(t) \text{ is } M_{i1} \text{ and } x_2(t) \text{ is } M_{i2} \cdots x_n(t) \text{ is } \\ & M_{in} \\ & \text{ THEN} \\ & \dot{x}(t) = [A_i + \rho(A_i)]x(t) + [B_i + \rho(B_i)]u(t) \\ & \quad + G_i w(t), \quad t \neq t_k \\ & z(t) = C_i x(t) \end{aligned} \tag{4}$$

$$\Delta x = B_k x(t), \quad t = t_k$$

$$x(t_0^+) = x_0$$

• INPUT-OUTPUT form:

$$\begin{aligned} \dot{x}(t) &= \frac{\sum_{i=1}^r w_i [A_i + \rho(A_i)]x(t) + [B_i + \rho(B_i)]u(t) + G_i w(t)}{\sum_{i=1}^r w_i} \\ &= \sum_{i=1}^r h_i \{ [A_i + \rho(A_i)]x(t) \\ & \quad + [B_i + \rho(B_i)]u(t) + G_i w(t) \}, \quad t \neq t_k \\ z(t) &= \frac{\sum_{i=1}^r w_i C_i x(t)}{\sum_{i=1}^r w_i} = \sum_{i=1}^r h_i C_i x(t) \end{aligned} \tag{5}$$

$$\Delta x = \sum_{i=1}^r h_i B_{ik} x(t) := U(t, x(t)), \quad t = t_k$$

$$x(t_0^+) = x_0$$

$$w_i = \prod_{j=1}^n M_{ij}(x_j(t)), \quad \sum_{i=1}^r w_i > 0, \quad w_i > 0,$$

$$h_i = \frac{w_i}{\sum_{i=1}^r w_i}, \quad \sum_{i=1}^r h_i = 1, \quad h_i > 0.$$

where $x(t) = [x_1(t), x_2(t) \cdots x_n(t)]^T$ is state vector, $u(t) \in R^m$ is control input, $z(t) \in R^p$ is control output, A_i, B_i, G_i and C_i are constant matrices and of appropriate dimensions, $\rho(A_i), \rho(B_i)$ are unknown disturbance matrix which satisfy

$$\rho(A_i) = E_i \Lambda_i F_i, \|\rho(B_i)\| \leq r_0, \forall i \in \{1, 2, \dots, r\} \tag{6}$$

where Λ_i satisfy

$$\Lambda_i^T \Lambda_i \leq I, \quad \forall i \in \{1, 2, \dots, r\} \tag{7}$$

where

$$0 = t_0 < t_1 < t_2 < \dots < t_k \cdots, \lim_{k \rightarrow \infty} t_k = \infty$$

and

$$\Delta x|_{t_k} := x(t_k^+) - x(t_k^-)$$

where $x(t_k^+) = \lim_{h \rightarrow 0^+} x(t_k + h)$ and $x(t_k^-) = \lim_{h \rightarrow 0^+} x(t_k - h)$,

For simplicity it is assumed that, $x(t_k^-) = x(t_k)$, $x_0 = 0$ and

$$B_k = bI, \quad -2 \leq b \leq 0 \quad (k = 1, 2, \dots)$$

Thus we can easily obtain $U(t_k, x(t_k)) \equiv b x(t_k)$.

For the nonlinear plant represented by (4) or (5), we consider the fuzzy controller as follows:

• IF-THEN form:

$$\begin{aligned} R_i : & \text{ IF } x_1(t) \text{ is } M_{i1} \text{ and } x_2(t) \text{ is } M_{i2} \cdots x_n(t) \text{ is } \\ & M_{in} \\ & \text{ THEN} \\ & u(t) = L_i q_\mu(x) \end{aligned} \tag{8}$$

• INPUT-OUTPUT form:

$$u(t) = \sum_{i=1}^r h_i^{q_\mu(x)} [L_i q_\mu(x)] \tag{9}$$

The system (5) with (9) can be written the form of the T-S fuzzy control system as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i,j=1}^r h_i h_j^{q_\mu(x)} \{ [H_{ij} + \rho(A_i) + \rho(B_i)L_j]x(t) \\ &+ G_i w(t) + [B_i + \rho(B_i)]L_j \mu [q(\frac{x}{\mu}) - \frac{x}{\mu}] \}, t \neq t_k \\ z(t) &= \frac{\sum_{i=1}^r w_i C_i x(t)}{\sum_{i=1}^r w_i} = \sum_{i=1}^r h_i C_i x(t) \end{aligned} \tag{10}$$

$$\Delta x = U(t, x(t)) = bx(t), \quad t = t_k$$

$$x(t_0^+) = x_0$$

where H_{ij} denotes $H_{ij} = A_i + B_i L_j$.

Our main aim is to find some sufficient condition which make the closed-loop system (10) exponentially stable and L_2 gain less than or equal to γ , we require the following definitions:

Definition 2.1: Suppose γ is given positive real number. The uncertain impulsive system (10) is said to have L_2 gain less than or equal to γ (L_2 gain $\leq \gamma$) if

$$\int_0^\infty z^T(t)z(t)dt \leq \gamma^2 \int_0^\infty w^T(t)w(t)dt. \tag{11}$$

Definition 2.2: The uncertain impulsive system (10) is said to be exponentially stable with an H^∞ norm γ if

(1) (Internal exponential stability) Systems (10) with $w(t) = 0$ ($\forall t \in R$), the trivial solution (equilibrium point) is exponentially stable;

(2) (L_2 gain $\leq \gamma$) Systems (10) have L_2 gain less than or equal to γ .

III. STATE FEEDBACK CONTROL

In this section, for the nonlinear plant represented by (4) or (5), we consider the fuzzy non-quantized feedback controller as follows:

• IF-THEN form:

$$R_i : \text{IF } x_1(t) \text{ is } M_{i1} \text{ and } x_2(t) \text{ is } M_{i2} \cdots x_n(t) \text{ is } M_{in}$$

THEN

$$u(t) = L_i x(t) \tag{12}$$

• INPUT-OUTPUT form:

$$u(t) = \sum_{i=1}^r h_i [L_i x(t)] \tag{13}$$

The system (5) with (13) can be written the form of the T-S fuzzy control system as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i,j=1}^r h_i h_j \{ [H_{ij} + \rho(A_i) + \rho(B_i)L_j]x(t) \\ &+ G_i w(t) \}, t \neq t_k \end{aligned}$$

$$z(t) = \frac{\sum_{i=1}^r w_i C_i x(t)}{\sum_{i=1}^r w_i} = \sum_{i=1}^r h_i C_i x(t) \tag{14}$$

$$\Delta x = U(t, x(t)) = bx(t), \quad t = t_k$$

$$x(t_0^+) = x_0$$

where H_{ij} denotes $H_{ij} = A_i + B_i L_j$.

Consider H^∞ feedback control of system (14), we require the following Assumption 3.1:

Assumption 3.1: Assume that there exist a positive constant $\gamma > 0$, two sequences of constants $\{\eta_i\}_{i=1}^r, \{\eta_{i,j}\}_{i,j=1}^r$, a sequence of matrices $\{L_i\}_{i=1}^r$ and a common positive definite matrix P such that the sequence of matrices $\{Q_{ij}\}_{i,j=1}^r$ is positive and satisfies

$$\begin{aligned} Q_{ij} &:= -[(A_i + B_i L_j)^T P + P(A_i + B_i L_j) \\ &+ 2\lambda P + C_i^T C_i + \varepsilon I + \frac{1}{\gamma^2 - \varepsilon} P G_i G_i^T P \\ &+ \eta_i P E_i E_i^T P + \frac{1}{\eta_i} F_i^T F_i + \eta_{i,j} P^2 + \frac{r_0^2}{\eta_{i,j}} L_j^T L_j] \\ &= -[H_{ij}^T P + P H_{ij} + 2\lambda P \\ &+ C_i^T C_i + \varepsilon I + \frac{1}{\gamma^2 - \varepsilon} P G_i G_i^T P \\ &+ \eta_i P E_i E_i^T P + \frac{1}{\eta_i} F_i^T F_i + \eta_{i,j} P^2 + \frac{r_0^2}{\eta_{i,j}} L_j^T L_j]. \end{aligned} \tag{15}$$

Theorem 3.2: Assume there exist nonnegative constant ε and γ (L_2 gain $\leq \gamma$) such that Assumption 3.1 holds, then the closed-loop system (14) is exponentially stable with an H^∞ norm γ .

In order to prove Theorem 3.2, we require the following Lemma 3.3:

Lemma 3.3^[3]: Assume X, Y, Z is constant real matrix of appropriate dimension, satisfying $Y^T Y \leq I$, then $\forall \eta > 0$, we have

$$XYZ + Z^T Y^T X^T \leq \eta Z^T Z + \frac{1}{\eta} X X^T.$$

Proof of Theorem 3.2: We consider the Lyapunov function candidate $V(x) = x^T P x$ for the closed-loop system (14), the derivative of $V(x)$ along solutions of (14) in $t \in (t_{k-1}, t_k]$ is computed as

$$\begin{aligned} \dot{V}(x) &= \sum_{i,j=1}^r h_i h_j \{ x^T(t) (H_{ij}^T P + P H_{ij}) x(t) \\ &+ 2x^T(t) (\rho(A_i) + \rho(B_i)L_j) x(t) \\ &+ 2x^T(t) P G_i w(t) \} \end{aligned} \tag{16}$$

By Lemma 3.3, we obtain

$$\begin{aligned} 2x^T P \rho(A_i)x &= 2x^T P E_i \Lambda_i F L_i x \\ &\leq \eta_i x^T P E_i E_i^T P x + \frac{1}{\eta_i} x^T F_i^T F_i x \\ &\quad 2x^T P \rho(B_i) L_j x \\ &\leq \eta_{i,j} x^T P^2 x + \frac{1}{\eta_{i,j}} x^T L_j^T \rho^T(B_i) \rho(B_i) L_j x \\ &\leq \eta_{i,j} x^T P^2 x + \frac{r_0^2}{\eta_{i,j}} x^T L_j^T L_j x \end{aligned}$$

Let

$$H(t) := \dot{V}(x) + 2\lambda V(x) + z^T z - \gamma^2 w^T w + \varepsilon(x^T x + w^T w)$$

We have

$$\begin{aligned} H(t)|_{(14)} &= \sum_{i,j=1}^r h_i h_j \{x^T(t)(H_{ij}^T P + P H_{ij} \\ &\quad + 2\lambda P + C_i^T C_i + \varepsilon I)x(t) \\ &\quad + 2x^T(t)[\rho(A_i) + \rho(B_i)L_j]x(t) \\ &\quad + 2x^T(t)P G_i w(t) - (\gamma^2 - \varepsilon)w^T w\} \end{aligned}$$

However

$$\begin{aligned} &2x^T(t)P G_i w(t) - (\gamma^2 - \varepsilon)w^T w \\ &= -(\gamma^2 - \varepsilon)\left[\left(w - \frac{1}{\gamma^2 - \varepsilon} G_i^T P x\right)^T \left(w - \frac{1}{\gamma^2 - \varepsilon} G_i^T P x\right)\right] \\ &\quad + \frac{1}{\gamma^2 - \varepsilon} x^T P G_i G_i^T P x \end{aligned}$$

Therefore, by (15), we obtain

$$\begin{aligned} &H(t)|_{(14)} \\ &= \sum_{i,j=1}^r h_i h_j \{x^T(t)(H_{ij}^T P + P H_{ij} \\ &\quad + 2\lambda P + C_i^T C_i + \varepsilon I + \frac{1}{\gamma^2 - \varepsilon} P G_i G_i^T P)x(t) \\ &\quad + 2x^T(t)[\rho(A_i) + \rho(B_i)L_j]x(t) \\ &\quad - (\gamma^2 - \varepsilon)\left[\left(w - \frac{1}{\gamma^2 - \varepsilon} G_i^T P x\right)^T \left(w - \frac{1}{\gamma^2 - \varepsilon} G_i^T P x\right)\right]\} \\ &\leq \sum_{i,j=1}^r h_i h_j \{x^T(t)(H_{ij}^T P + P H_{ij} + 2\lambda P \\ &\quad + C_i^T C_i + \varepsilon I + \frac{1}{\gamma^2 - \varepsilon} P G_i G_i^T P + \eta_i P E_i E_i^T P \\ &\quad + \frac{1}{\eta_i} F_i^T F_i + \eta_{i,j} P^2 + \frac{r_0^2}{\eta_{i,j}} L_j^T L_j)x(t) \\ &\quad - (\gamma^2 - \varepsilon)\left[\left(w - \frac{1}{\gamma^2 - \varepsilon} G_i^T P x\right)^T \left(w - \frac{1}{\gamma^2 - \varepsilon} G_i^T P x\right)\right]\} \end{aligned}$$

$$\begin{aligned} &\leq \sum_{i,j=1}^r h_i h_j \{-x^T(t)Q_{ij}x(t) \\ &\quad - (\gamma^2 - \varepsilon)\left[\left(w - \frac{1}{\gamma^2 - \varepsilon} G_i^T P x\right)^T \left(w - \frac{1}{\gamma^2 - \varepsilon} G_i^T P x\right)\right]\} \\ &\leq \sum_{i,j=1}^r h_i h_j \{-x^T(t)Q_{ij}x(t)\} \\ &\leq 0 \end{aligned}$$

That is to say

$$\begin{aligned} &\dot{V}(x) + 2\lambda V(x) + z^T z - \gamma^2 w^T w \\ &\leq -\varepsilon(x^T x + w^T w) \\ &\leq 0, \quad \forall t \in (t_{k-1}, t_k]. \end{aligned} \tag{17}$$

Hence, $\forall T \in (t_{k-1}, t_k]$, integrating (17), we have

$$\begin{aligned} &\int_0^T \dot{V}(x)|_{(14)} dt + 2\lambda \int_0^T V(x)|_{(14)} dt \\ &\quad + \int_0^T (z^T z - \gamma^2 w^T w) dt \\ &\leq 0 \end{aligned} \tag{18}$$

By virtue of $\Delta x|_{t_k} := x(t_k^+) - x(t_k^-) = b x(t_k)$, we have

$$x(t_k) = b x(t_k) + x(t_k) = (1 + b)x(t_k)$$

and

$$\begin{aligned} &V(x(t_k)) - V(x(t_k^-)) \\ &= x^T(t_k) P x(t_k) - x^T(t_k^-) P x(t_k^-) \\ &= x^T(t_k) P x(t_k) - (1 + b^2)x^T(t_k) P x(t_k) \\ &= -b(2 + b)V(x(t_k)) \end{aligned}$$

Hence

$$\begin{aligned} &\int_0^T \dot{V}(x)|_{(14)} dt \\ &= \int_0^{t_1} \dot{V}(x)|_{(14)} dt + \int_{t_1}^{t_2} \dot{V}(x)|_{(14)} dt \\ &\quad + \dots + \int_{t_{k-1}}^{t_k} \dot{V}(x)|_{(14)} dt + \int_{t_k}^T \dot{V}(x)|_{(14)} dt \\ &= V(x(t_1)) - V(x(0^+)) \\ &\quad + V(x(t_2)) - V(x(t_1^+)) + \dots \\ &\quad + V(x(t_k)) - V(x(t_{k-1}^+)) + V(x(T)) - V(x(t_k^+)) \\ &= V(x(t_1)) - V(x(t_1^+)) + \dots \\ &\quad + V(x(t_k)) - V(x(t_k^+)) + V(x(T)) \\ &= -\sum_{i=1}^k b(2 + b)V(x(t_i)) + V(x(T)) \\ &\geq 0 \end{aligned}$$

Therefore, we can obtain

$$\int_0^T (z^T z - \gamma^2 w^T w) dt \leq 0 \tag{19}$$

That is to say

$$\int_0^T z^T z dt \leq \gamma^2 \int_0^T w^T w dt \tag{20}$$

When $w = 0$, one has

$$\dot{V}(x) + 2\lambda V(x) \leq 0, \forall t \in (t_{k-1}, t_k]. \tag{21}$$

Hence, we can easily achieve that system (14) is exponentially stable with an H^∞ norm γ .

IV. QUANTIZED FEEDBACK CONTROL

Considering the H^∞ feedback control of nonlinear uncertain fuzzy impulsive systems (5) with quantized controller law (9), we have the following result:

Theorem 4.1: Assume there exist nonnegative constant ε and γ (L_2 gain $\leq \gamma$) such that Assumption 3.1 holds; Moreover, assume that for arbitrary fixed $\sigma > 0$, M is chosen large enough such that

$$M > \Theta_x \Delta (1 + \sigma) \tag{22}$$

where

$$\begin{aligned} \Theta_x &:= 2 \frac{\|P\| \|L\| (\|B\| + r_0)}{\bar{\lambda}} \\ \bar{\lambda} &:= \min \{ \lambda(Q_{ij}) : i, j = 1, 2, \dots, r \} \\ \|L\| &:= \max \{ \|L_j\| : j = 1, 2, \dots, r \} \\ \|B\| &:= \max \{ \|B_i\| : i = 1, 2, \dots, r \}. \end{aligned}$$

Then there exists a control strategy μ which depends on the state, and makes the closed-loop system (10) exponentially stable with an H^∞ norm γ .

Proof of Theorem 4.1: We consider the Lyapunov function candidate $V(x) = x^T P x$ for the closed-loop system (10), the derivative of $V(x)$ along solutions of (10) in $t \in (t_{k-1}, t_k]$ is computed as

$$\begin{aligned} \dot{V}(x) &= \sum_{i,j=1}^r h_i h_j^{q_\mu(x)} \{ x^T(t) (H_{ij}^T P + P H_{ij}) x(t) \\ &\quad + 2x^T(t) [\rho(A_i) + \rho(B_i) L_j] x(t) \\ &\quad + 2x^T(t) P G_i w(t) \\ &\quad + 2x^T(t) P [B_i + \rho(B_i)] L_j \mu [q(\frac{x}{\mu}) - \frac{x}{\mu}] \} \end{aligned} \tag{23}$$

We have

$$\begin{aligned} H(t)|_{(10)} &:= \sum_{i,j=1}^r h_i h_j^{q_\mu(x)} \{ x^T(t) (H_{ij}^T P + P H_{ij}) \\ &\quad + 2\lambda P + C_i^T C_i + \varepsilon I \} x(t) \\ &\quad + 2x^T(t) [\rho(A_i) + \rho(B_i) L_j] x(t) \\ &\quad + 2x^T(t) P G_i w(t) - (\gamma^2 - \varepsilon) w^T w \\ &\quad + 2x^T(t) P [B_i + \rho(B_i)] L_j \mu [q(\frac{x}{\mu}) - \frac{x}{\mu}] \} \end{aligned}$$

Therefore, by (15), we obtain

$$\begin{aligned} &H(t)|_{(10)} \\ &= \sum_{i,j=1}^r h_i h_j^{q_\mu(x)} \{ x^T(t) (H_{ij}^T P + P H_{ij}) \\ &\quad + 2\lambda P + C_i^T C_i + \varepsilon I + \frac{1}{\gamma^2 - \varepsilon} P G_i G_i^T P \} x(t) \\ &\quad + 2x^T(t) [\rho(A_i) + \rho(B_i) L_j] x(t) \\ &\quad - (\gamma^2 - \varepsilon) [(w - \frac{1}{\gamma^2 - \varepsilon} G_i^T P x)^T (w - \frac{1}{\gamma^2 - \varepsilon} G_i^T P x)] \\ &\quad + 2x^T(t) P [B_i + \rho(B_i)] L_j \mu [q(\frac{x}{\mu}) - \frac{x}{\mu}] \} \\ &\leq \sum_{i,j=1}^r h_i h_j \{ -x^T(t) Q_{ij} x(t) \\ &\quad + 2x^T(t) P [B_i + \rho(B_i)] L_j \mu [q(\frac{x}{\mu}) - \frac{x}{\mu}] \\ &\quad - (\gamma^2 - \varepsilon) [(w - \frac{1}{\gamma^2 - \varepsilon} G_i^T P x)^T (w - \frac{1}{\gamma^2 - \varepsilon} G_i^T P x)] \} \\ &\leq \sum_{i,j=1}^r h_i h_j^{q_\mu(x)} \{ -x^T(t) Q_{ij} x(t) \\ &\quad + 2x^T(t) P [B_i + \rho(B_i)] L_j \mu [q(\frac{x}{\mu}) - \frac{x}{\mu}] \} \\ &\leq \sum_{i,j=1}^r h_i h_j^{q_\mu(x)} \{ -\bar{\lambda} |x| (|x| - \Theta_x \mu \Delta) \} \\ &= -\bar{\lambda} |x| (|x| - \Theta_x \mu \Delta) \end{aligned}$$

According to (22), for any nonzero x , we can find a positive scalar μ such that

$$\Theta_x (1 + \sigma) \mu \Delta \leq |x| \leq M \mu \tag{24}$$

This is also true in the case of $x = 0$, where we set μ as an extreme case and consider the output of the quantizer as zero.

Hence, there exists a control strategy for μ , such that $H(t)|_{(10)} \leq 0$. Repeating the approach of the above proof of Theorem 3.2, we can achieve the fuzzy impulsive system (10) exists a control strategy for μ such that the closed-loop system (10) is exponentially stable with an H^∞ norm γ .

V. FUZZY NONLINEAR IMPULSIVE SYSTEMS WITH NONLINEAR UNCERTAINTIES

Consider a nonlinear dynamic multi-input-multi-output system with nonlinear uncertainty modeled by the T-S fuzzy system, which can be represented by the following forms:

• IF-THEN form:

$$\begin{aligned} R_i : &\text{IF } x_1(t) \text{ is } M_{i1} \text{ and } x_2(t) \text{ is } M_{i2} \cdots x_n(t) \text{ is } \\ &M_{in} \\ \text{THEN} \\ &\dot{x}(t) = A_i x(t) + B_i u(t) \end{aligned}$$

$$\begin{aligned}
 &+G_i w(t) + f_i(x, t), \quad t \neq t_k \\
 z(t) &= C_i x(t) \tag{25}
 \end{aligned}$$

$$\Delta x = B_k x(t), \quad t = t_k$$

$$x(t_0^+) = x_0$$

• INPUT-OUTPUT form:

$$\begin{aligned}
 \dot{x}(t) &= \frac{\sum_{i=1}^r w_i [A_i x(t) + B_i u(t) + G_i w(t) + f_i(x, t)]}{\sum_{i=1}^r w_i} \\
 &= \sum_{i=1}^r h_i [A_i x(t) + B_i u(t) \\
 &\quad + G_i w(t) + f_i(x, t)], \quad t \neq t_k
 \end{aligned}$$

$$z(t) = \frac{\sum_{i=1}^r w_i C_i x(t)}{\sum_{i=1}^r w_i} = \sum_{i=1}^r h_i C_i x(t) \tag{26}$$

$$\Delta x = \sum_{i=1}^r h_i B_{ik} x(t) := U(t, x(t)), \quad t = t_k$$

$$x(t_0^+) = x_0$$

$$w_i = \prod_{j=1}^n M_{ij}(x_j(t)), \quad \sum_{i=1}^r w_i > 0, \quad w_i > 0,$$

$$h_i = \frac{w_i}{\sum_{i=1}^r w_i}, \quad \sum_{i=1}^r h_i = 1, \quad h_i > 0.$$

where $f_i(x, t)$ is nonlinear component, the other variables and impulsive properties are the same as system (4). We consider $f_i(x, t)$ satisfies

$$f_i^T(x, t) f_i(x, t) \leq x^T g(t) x, \quad \forall i \in \{1, 2, \dots, r\} \tag{27}$$

where $g(t)$ is known continuous function. We will give the sufficient conditions of the H^∞ control (26).

For the nonlinear plant represented by (25) or (26), we consider the fuzzy controller as follows:

• IF-THEN form:

R_i : IF $x_1(t)$ is M_{i1} and $x_2(t)$ is $M_{i2} \dots x_n(t)$ is M_{in}
THEN

$$u(t) = L_i x(t) \tag{28}$$

or

$$u(t) = L_i q_\mu(x) \tag{29}$$

• INPUT-OUTPUT form:

$$u(t) = \sum_{i=1}^r h_i [L_i x(t)] \tag{30}$$

or

$$u(t) = \sum_{i=1}^r h_i^{q_\mu(x)} [L_i q_\mu(x)] \tag{31}$$

The system (26) with (30) or (26) with (31) can be written the form of the T-S fuzzy control system as follows:

$$\begin{aligned}
 \dot{x}(t) &= \sum_{i,j=1}^r h_i h_j [(A_i + B_i L_j) x(t) + G_i w(t) + f_i(x, t)] \\
 &= \sum_{i,j=1}^r h_i h_j [H_{ij} x(t) + G_i w(t) + f_i(x, t)], \quad t \neq t_k
 \end{aligned}$$

$$z(t) = \frac{\sum_{i=1}^r w_i C_i x(t)}{\sum_{i=1}^r w_i} = \sum_{i=1}^r h_i C_i x(t) \tag{32}$$

$$\Delta x = U(t, x(t)) = b x(t), \quad t = t_k$$

$$x(t_0^+) = x_0$$

or

$$\begin{aligned}
 \dot{x}(t) &= \sum_{i,j=1}^r h_i h_j^{q_\mu(x)} \{ (A_i + B_i L_j) x(t) + G_i w(t) + f_i(x, t) \\
 &\quad + B_i L_j \mu [q(\frac{x}{\mu}) - \frac{x}{\mu}] \} \\
 &= \sum_{i,j=1}^r h_i h_j^{q_\mu(x)} \{ H_{ij} x(t) + G_i w(t), \\
 &\quad + f_i(x, t) + B_i L_j \mu [q(\frac{x}{\mu}) - \frac{x}{\mu}] \}, \quad t \neq t_k \tag{33}
 \end{aligned}$$

$$z(t) = \frac{\sum_{i=1}^r w_i C_i x(t)}{\sum_{i=1}^r w_i} = \sum_{i=1}^r h_i C_i x(t)$$

$$\Delta x = U(t, x(t)) = b x(t), \quad t = t_k$$

$$x(t_0^+) = x_0$$

where H_{ij} denotes $H_{ij} = A_i + B_i L_j$.

Consider H^∞ feedback control of system (33), we require the following Assumption 5.1:

Assumption 5.1: Assume that there exist a positive constant $\gamma > 0$, two sequences of constants $\{\eta_i\}_{i=1}^r$, a sequence of matrices $\{L_i\}_{i=1}^r$ and a common positive definite matrix P such that the sequence of matrices $\{\bar{Q}_{ij}(t)\}_{i,j=1}^r$ is positive and satisfies

$$\begin{aligned}
 \bar{Q}_{ij}(t) &:= -[(A_i + B_i L_j)^T P + P(A_i + B_i L_j) \\
 &\quad + 2\lambda P + C_i^T C_i \\
 &\quad + (\varepsilon + \frac{1}{\eta_i} g(t)) I + \eta_i P^2 + \frac{1}{\gamma^2 - \varepsilon} P G_i G_i^T P] \\
 &= -[H_{ij}^T P + P H_{ij} + 2\lambda P + C_i^T C_i \\
 &\quad + (\varepsilon + \frac{1}{\eta_i} g(t)) I + \eta_i P^2 + \frac{1}{\gamma^2 - \varepsilon} P G_i G_i^T P]. \tag{34}
 \end{aligned}$$

Theorem 5.1: Assume there exist nonnegative constant ε and γ (L_2 gain $\leq \gamma$) such that Assumption 5.1 holds, then the closed-loop system (32) is exponentially stable with an H^∞ norm γ .

Proof of Theorem 5.1: We consider the Lyapunov function candidate $V(x) = x^T P x$ for the closed-loop system (32), the derivative of $V(x)$ along solutions of (32) in $t \in (t_{k-1}, t_k]$ is computed as

$$\dot{V}(x) = \sum_{i,j=1}^r h_i h_j \{ x^T(t) (H_{ij}^T P + P H_{ij}) x(t) \}$$

$$+2x^T(t)Pf_i(x,t) + 2x^T(t)PG_iw(t)\} \quad (35)$$

By Lemma 3.3, we obtain

$$2x^T(t)Pf_i(x,t) \leq x^T[\eta_i P^2 + \frac{1}{\eta_i}g(t)I]x$$

Let

$$H(t) := \dot{V}(x) + 2\lambda V(x) + z^T z - \gamma^2 w^T w + \varepsilon(x^T x + w^T w)$$

Therefore, by (34), we obtain

$$\begin{aligned} & H(t)|_{(32)} \\ & \leq \sum_{i,j=1}^r h_i h_j \{x^T(t)(H_{ij}^T P + PH_{ij} + 2\lambda P + C_i^T C_i \\ & \quad + (\varepsilon + \frac{1}{\eta_i}g(t))I + \eta_i P^2 + \frac{1}{\gamma^2 - \varepsilon}PG_i G_i^T P)x(t) \\ & \quad - (\gamma^2 - \varepsilon)[(w - \frac{1}{\gamma^2 - \varepsilon}G_i^T Px)^T (w - \frac{1}{\gamma^2 - \varepsilon}G_i^T Px)]\} \\ & = \sum_{i,j=1}^r h_i h_j \{-x^T(t)\bar{Q}_{ij}(t)x(t) \\ & \quad - (\gamma^2 - \varepsilon)[(w - \frac{1}{\gamma^2 - \varepsilon}G_i^T Px)^T (w - \frac{1}{\gamma^2 - \varepsilon}G_i^T Px)]\} \\ & \leq \sum_{i,j=1}^r h_i h_j \{-x^T(t)\bar{Q}_{ij}(t)x(t)\} \\ & \leq 0 \end{aligned}$$

Hence repeating the approach of the above proof of Theorem 3.2, we can achieve the system (26) with controller law (30) exists a control strategy for μ such that the closed-loop system (32) is exponentially stable with an H^∞ norm γ .

Theorem 5.2: Assume there exist nonnegative constant ε and γ (L_2 gain $\leq \gamma$) such that Assumption 5.1 holds; Moreover, assume that for arbitrary fixed $\sigma > 0$, M is chosen large enough such that

$$M > \bar{\Theta}_x \Delta (1 + \sigma) \quad (36)$$

where

$$\begin{aligned} \bar{\Theta}_x & := \frac{2\theta}{\hat{\lambda}} \\ \hat{\lambda} & := \min\{\lambda(\bar{Q}_{ij}(t)) : i, j = 1, 2, \dots, r; t \geq 0\} \\ \theta & := \max\{\|PB_i L_j\| : i, j = 1, 2, \dots, r\}. \end{aligned}$$

Then there exists a control strategy μ which depends on the state, and makes the closed-loop system (33) exponentially stable with an H^∞ norm γ .

Proof of Theorem 5.2: We consider the Lyapunov function candidate $V(x) = x^T Px$ for the closed-loop system (33), the derivative of $V(x)$ along solutions of (33) in $t \in (t_{k-1}, t_k]$ is computed as

$$\begin{aligned} \dot{V}(x) & = \sum_{i,j=1}^r h_i h_j^{q_\mu(x)} \{x^T(t)(H_{ij}^T P + PH_{ij})x(t) \\ & \quad + 2x^T(t)Pf_i(x,t) + 2x^T(t)PG_iw(t) \\ & \quad + 2x^T(t)PB_i L_j \mu [q(\frac{x}{\mu}) - \frac{x}{\mu}]\} \end{aligned} \quad (37)$$

Let

$$H(t) := \dot{V}(x) + 2\lambda V(x) + z^T z - \gamma^2 w^T w + \varepsilon(x^T x + w^T w)$$

We have

$$\begin{aligned} & H(t)|_{(33)} \\ & \leq \sum_{i,j=1}^r h_i h_j^{q_\mu(x)} \{x^T(t)[H_{ij}^T P + PH_{ij} \\ & \quad + 2\lambda P + C_i^T C_i + (\varepsilon + \frac{1}{\eta_i}g(t))I + \eta_i P^2]x(t) \\ & \quad + 2x^T(t)PG_iw(t) - (\gamma^2 - \varepsilon)w^T w \\ & \quad + 2x^T(t)PB_i L_j \mu [q(\frac{x}{\mu}) - \frac{x}{\mu}]\} \end{aligned}$$

According to (36), we can find a positive scalar μ such that

$$\bar{\Theta}_x (1 + \sigma) \mu \Delta \leq |x| \leq M \mu \quad (38)$$

Therefore, by (34), we obtain

$$\begin{aligned} & H(t)|_{(33)} \\ & \leq \sum_{i,j=1}^r h_i h_j^{q_\mu(x)} \{x^T(t)[H_{ij}^T P + PH_{ij} + 2\lambda P + C_i^T C_i \\ & \quad + (\varepsilon + \frac{1}{\eta_i}g(t))I + \eta_i P^2 + \frac{1}{\gamma^2 - \varepsilon}PG_i G_i^T P]x(t) \\ & \quad - (\gamma^2 - \varepsilon)[(w - \frac{1}{\gamma^2 - \varepsilon}G_i^T Px)^T (w - \frac{1}{\gamma^2 - \varepsilon}G_i^T Px)] \\ & \quad + 2x^T(t)PB_i L_j \mu [q(\frac{x}{\mu}) - \frac{x}{\mu}]\} \\ & = \sum_{i,j=1}^r h_i h_j^{q_\mu(x)} \{-x^T(t)\bar{Q}_{ij}(t)x(t) \\ & \quad - (\gamma^2 - \varepsilon)[(w - \frac{1}{\gamma^2 - \varepsilon}G_i^T Px)^T (w - \frac{1}{\gamma^2 - \varepsilon}G_i^T Px)] \\ & \quad + 2x^T(t)PB_i L_j \mu [q(\frac{x}{\mu}) - \frac{x}{\mu}]\} \\ & \leq \sum_{i,j=1}^r h_i h_j^{q_\mu(x)} \{-x^T(t)\bar{Q}_{ij}(t)x(t) \\ & \quad + 2x^T(t)PB_i L_j \mu [q(\frac{x}{\mu}) - \frac{x}{\mu}]\} \\ & \leq -\hat{\lambda} |x| (|x| - \bar{\Theta}_x \mu \Delta) \leq 0 \end{aligned}$$

Hence, there exists a control strategy for μ , such that $H(t)|_{(33)} \leq 0$. Repeating the approach of the above proof of Theorem 3.2, we can achieve the fuzzy impulsive system (33)

exists a control strategy for μ such that the closed-loop system (33) is exponentially stable with an H^∞ norm γ .

VI. CONCLUSIONS

In this paper, we discuss the problem of H^∞ control of two classes of fuzzy nonlinear impulsive systems with quantized feedback. New results on the H^∞ control problems are established for one class of fuzzy nonlinear uncertain impulsive systems and one class of fuzzy nonlinear impulsive systems with nonlinear uncertainties by choosing appropriately quantized strategies, respectively.

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