DSA Image Fusion Based on Dynamic Fuzzy Logic and Curvelet Entropy

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***Abstract*—The curvelet transform as a multiscale transform has directional parameters occurs at all scales, locations, and orientations. It is superior to wavelet transform in image processing domain. This paper analyzes the characters of DSA medical image, and proposes a novel approach for DSA medical image fusion, which is using curvelet information entropy and dynamic fuzzy logic. Firstly, the image was decomposed by curvelet transform to obtain the different level information. Then the entropy from different level of DSA medical image was calculated, and a membership function based on dynamic fuzzy logic was constructed to adjust the weight for image subbands coefficients via entropy. At last an inverse curvelet transform was applied to reconstruct the image to synthesize one DSA medical image which could contain more integrated accurate detail information of blood vessels than any one of the individual source images. By compare, the efficiency of our method is better than weighted average, laplacian pyramid and traditional wavelet transform method.**

***Index Terms*—curvelet transform, DSA image fusion, entropy, dynamic fuzzy logic**

I. Introduction

In recent years, the curvelet transform has received more and more attention due to its particular characters. Many Classical multiresolution ideas only address a portion of the whole range of possible multiscale phenomena, like the classical wavelet viewpoint, there are objects, e.g. images that do not exhibit isotropic scaling and, thus, call for other types of multiscale representation. Candes and Donoho [1] develeped a new theory of multiresolution analysis called the curvelet transform. This mathematical transform differs from wavelet and related other mathematical transform. Curvelets take the form of basis elements, which exhibit a very high directional sensitivity and are highly anisotropic. In two dimensions, for instance, curvelets are localized along curves and in three dimensions along sheets. Because this new mathematic transform is based on the wavelet transform and radon transform. It has overcome some limitations of wavelet transform in medical image fusion.

Digital subtraction angiography has been widely used in both medical diagnoses and interposal therapy. Digital subtraction angiography [2], an electronic technique for imaging blood vessels, is useful in diagnosing arterial occlusion, including carotid artery stenosis and pulmonary artery thrombosis, and in detecting renal vascular disease. The images are all produced in real time by the computer. Firstly, we acquire images which contain all tissues by exposing the area of interest with time-controlled x-ray energy. We define an image without contrast material as mask image. Secondly, a physician injected contrast material into an artery or vein to the same area to produce fluoroscopic image, which we define it as actual angiographic image. Finally, using these digitized images, a computer subtracts the mask image from the actual angiographic image, producing an image that only allows the contrast material in the arteries to be seen more clearly. In this manner, the soft tissues, bones and gas were same in the initial image, but they were eliminated in the DSA medical image by the subtraction processing. The remaining images of blood vessels containing the contrast material are thus more prominent, all the overlying background is taken away.

The images of the source angiographic subtraction image and the DSA medical image after subtraction are shown in Figure 1 and Figure 2.

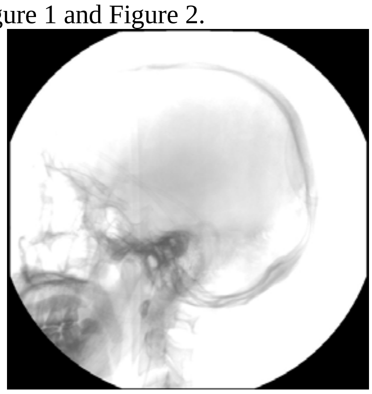


Figure 1. Source Angiographic Subtraction Image

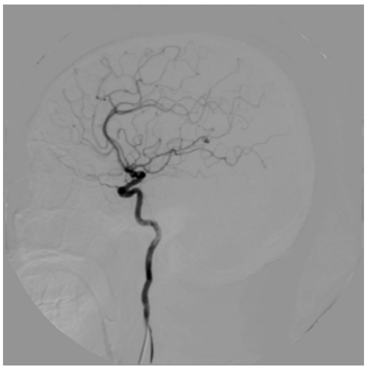


Figure 2. DSA Medical Image after Subtraction

By analyzing the characters of DSA medical image sequence, we find that the background image and each angiogram are taken by the same instrument at different time. Because the dye inject into the blood vessels is flowing, dissolving and disappearing as time gose by, after subtraction, we obtain the DSA serial images are present different parts of blood vessels. The goal of image fusion is to synthesize information from multiple images of the same scene to one composite image that contains more accurate descriptive information of the scene than any one of the individual source images [3,4]. Thus we fuse these serial images into one image which contain whole image of blood vessels. It is useful for medical experts to locate interesting region accurately. That will become a novel approach of medical image processing.

Image fusion has become an important and useful technique for image analysis and computer vision, medical diagnosis, remote sensing, concealed weapon detection, and night vision applications. In recent years, various image fusion methods have been proposed and researched such as the weighted average method, laplacian pyramid method and wavelet transform method etc [5,6]. Due to the fuzzy characters of the information that DSA medical serial images contain, this paper presents a novel image fusion methodology, which is using digital curvelet transform and dynamic fuzzy logic.

There are a great number of developments and a series of substantial achievements in the domain of fuzzy mathematics' theory research and application, since L. A. Zadeh proposed fuzzy sets in 1965 [7]. However, theses theories can only help to solve those static problems. Dynamic fuzzy logic as an effective theory to solve dynamic fuzzy problems is widely researched. In real world, dynamic fuzzy problems exist universally, especially in the domain of image fusion. For example, these images become smoother and smoother. The word "become" reflects dynamic character and "smoother" reflects fuzzy character. The whole clause is dynamic fuzzy data. Dynamic fuzzy logic (DFL) based on dynamic fuzzy data is used to solve those problems and has made a series of research achievements. Because of the fuzzy character of the image fusion coefficients, we apply of DFL to analysis the curvelet entropy for medical image fusion.

Curvelet transform as a newly developed mathematical transform is often used as time-frequency and multiresolution analysis tool in the signal and image processing domain. It combined the anisotropic of ridgelet with the multiscale characteristic of wavelet. The prominent characteristic of curvelet is multiscale and high anisotropic, the curvelet transform is well-adapted to analyze and synthesize medical images containing edges. In view of the combination curvelet transform technique and DFL theory, This research is initial. For this reason, a DSA medical image fusion based on curvelet transform and DFL are proposed in the paper.

The rest of this paper is organized as follows. The curvelet transform analysis is given in Sections II. Dynamic fuzzy logic theory is given in Sections III. Section IV proposes curvelet fusion by entropy and DFL. Section V describes our method in the experiment and discusses the results. Conclusions are presented in last section.

II. Curvelet Transform Analysis

A new multi-scale transform which they called the curvelet transform [1] was developed by Cand'es and Donoho recently. For image analysis, it was nevertheless first proposed in the context of objects f(xl; x2) defined on the continuum plane(x1,x2) $\in R^2$. The transform was designed to represent edges and other singularities along curves much more efficiently than traditional transforms, i.e. using fewer coefficients for a given accuracy of reconstruction. That represent an edge to squared error 1/N requires 1/N wavelets and only about $1/\sqrt{N}$ curvelets.

The curvelet transform, like the wavelet transform, is a multiscale transform, with frame elements indexed by scale and location parameters. Unlike the wavelet transform, it has directional parameters, and the curvelet pyramid contains elements with a very high degree of directional specificity. In addition, the curvelet transform is based on a certain anisotropic scaling principle which is quite different from the isotropic scaling of wavelets. The elements obey a special scaling law, where the length of the support of frame elements and the width of the support are linked by the relation:

$$\text{width} \approx \text{length}^2$$

Curvelet is based on combining several ideas, which are briefly reviewed:

- Ridgelets, a method of analysis suitable for objects with discontinuities across straight lines.
- Multiscale Ridgelets, a pyramid of windowed ridgelets, renormalized and transported to a wide range of scales and locations.
- Bandpass filtering, a method of separating an object out into a series of disjoint scales.

A. Ridgelet Transform

We define an integrable bivariate function $f(x) \in R^2$ relative. The continuous ridgelet transform (CRT) [8, 9] in R^2 is defined as follows:

$$CRT_f(a,b,\theta) = \int_{R^2} \psi_{a,b,\theta}(x) f(x) dx \qquad (1)$$

where the ridgelets $\psi_{a,b,\theta}(x)$ in 2-D are defined from a wavelet-type function in 1-D $\psi(x)$ as follows:

$$\psi_{a,b,\theta}(x) = a^{\frac{1}{2}} \psi\left(\frac{x_1 \cos(\theta) + x_2 \sin(\theta) - b}{a}\right) \qquad (2)$$

The (separable) continuous wavelet transform (CWT) in R^2 of $f(x)$ can be written as follows:

$$CWT_f(a_1,a_2,b_1,b_2) = \int_{R^2} \psi_{a_1,a_2,b_1,b_2}(x) f(x) dx \qquad (3)$$

where the wavelets in 2-D are tensor products

$$\psi_{a_1,a_2,b_1,b_2}(x) = \psi_{a_1,b_1}(x_1)\psi_{a2,b2}(x_2) \quad (4)$$

of 1-D wavelets, $\psi_{a,b}(t) = a^{\frac{1}{2}}\psi\left(\frac{t-b}{a}\right)$.

By comparison, we can see, the CRT is similar to the 2-D CWT except that the point parameters (b_1, b_2) are replaced by the line parameters (b, θ). That is to say, these 2-D multiscale transforms have the relations as follows:

$$\text{Wavelets} \rightarrow \psi_{\text{scale,point-position}}$$

$$\text{Ridgelets} \rightarrow \psi_{\text{scale,line-position}}$$

Therefore, wavelets are very effective in representing objects with isolated point singularities, while ridgelets are very effective in representing objects with singularities along lines. In fact, one can think of ridgelets as a way of concatenating 1-D wavelets along lines. Hence the motivation for using ridgelets in image processing tasks is appealing since singularities are often joined together along edges or contours in images.

It is easy to extend the 1-D case to the 2-D case, points and lines are related via the Radon transform, thus the wavelet and ridgelet transforms are linked via the Radon transform. More precisely, denote the Radon transform as follows:

$$R_f(\theta,t) = \int_{R^2} f(x)\psi(x_1\cos\theta + x_2\sin\theta - t)dx \quad (5)$$

then the ridgelet transform is the application of a 1-D wavelet transform to the slices (also referred to as projections) of the Radon transform,

$$CWT_f(a,b,\theta) = \int_{R^2}\psi_{a,b}(t)R_f(\theta,t)dt \quad (6)$$

B. Curvelet Transform

Generally speaking, curvelet transform extends the ridgelet transform to multiple scale analysis. This means that ridgelet can be tuned to different orientations and different scales to create the curvelets, It is in the similar to Gabor filters. But different from Gabor filters which only cover part of the spectrum in the frequency domain [10], curvelets have a complete cover of the spectrum in frequency domain. That means, there is no loss of information in curvelet transform in terms of fusing the frequency information from images.

The curvelet transform opens us the possibility to analyse an image with different block sizes, but with a single transform. The idea is to first decompose the image into a set of wavelet bands, and to analyze each band by a ridgelet transform. The block size can be changed at each scale level.

The curvelet tight frame for L^2 (R^2) is a collection of analyzing elements γ_μ = γ_μ (x1, x2) indexed by tuples $\mu \in M'$ to be described below. It has been defined in [11] and has the following key properties:

- Transform Definition:

$$\alpha_\mu \equiv \langle f, \gamma_\mu \rangle, \quad \mu \in M' \quad (7)$$

- Parseval Relation:

$$\|f\|_2^2 = \sum_{\mu \in M'} |\alpha_\mu|^2 \quad (8)$$

- L^2 Reconstruction Formula:

$$f = \sum_{\mu \in M'} \langle f, \gamma_\mu \rangle \gamma_\mu \quad (9)$$

These formal properties are very similar to those one expects from an orthonormal basis, and reflect an underlying stability of representation.

The Curvelet Transform includes four stages:

(1) Sub-band decomposition:

$$f \mapsto (P_0 f, \Delta_1 f, \Delta_2 f, \ldots) \quad (10)$$

(2) Smooth partitioning:

$$h_Q = w_Q \cdot \Delta_s f \quad (11)$$

A grid of dyadic squares is defined as follows:

$$Q_{(s,k_1,k_2)} = \left[\frac{k_1}{2^s}, \frac{k_1+1}{2^s}\right] \times \left[\frac{k_2}{2^s}, \frac{k_2+1}{2^s}\right] \in \mathbf{Q}_s \quad (12)$$

Q*s* – all the dyadic squares of the grid.

(3) Renormalization:

$$(T_Q f)(x_1, x_2) = 2^s f(2^s x_1 - k_1, 2^s x_2 - k_2) \quad (13)$$

$$g_Q = T_Q^{-1} h_Q \quad (14)$$

(4) Ridgelet analysis:

$$\alpha_{(Q,\lambda)} = \langle g_Q, \rho_\lambda \rangle \quad (15)$$

There is also procedural definition of the reconstruction algorithm. The Inverse of the Curvelet Transform:

(1) Ridgelet Synthesis:

$$g_Q = \sum_\lambda \alpha_{(Q,\lambda)} \cdot \rho_\lambda \quad (16)$$

(2) Renormalization:

$$h_Q = T_Q g_Q \quad (17)$$

(3) Smooth Integration:

$$\Delta_s f = \sum_{Q \in \mathbf{Q}_s} w_Q \cdot h_Q \quad (18)$$

(4) Sub-band Recomposition:

$$f = P_0(P_0 f) + \sum_s \Delta_s(\Delta_s f) \quad (19)$$

Curvelet transform is defined via above concepts.

III. DYNAMIC FUZZY LOGIC (DFL)

In order to apply dynamic fuzzy logic theory in the domain of image fusion, we introduce the dynamic fuzzy logic concepts, dynamic fuzzy data, then propose estimate of dynamic fuzzy data and DFL proposition calculus[12].

A. Dynamic Fuzzy Data

Several universal accepted definitions was provided before the calculation of dynamic fuzzy data is given.

Pact 1. The character of data with both dynamic and fuzzy is called dynamic fuzzy character.

Example 1. She becomes more and more beautiful. The word “becomes” reflect the dynamic character and “beautiful” reflect the fuzzy character.

Pact 2. The data with the dynamic fuzzy character is called dynamic fuzzy data.

Example 2. It becomes warmer and warmer. The word "becomes" reflects the dynamic character and "warmer" reflects the fuzzy character. Then we call the whole clause as dynamic fuzzy data.

We based calculation of dynamic fuzzy data on the theory of dynamic fuzzy sets.

Pact 3. We call the collect of dynamic fuzzy data as dynamic fuzzy data sets.

Definition 1. Let a mapping be defined in the domain U.

$\overrightarrow{A}:\vec{a}\rightarrow[0,1]$, $\vec{a}\rightarrow\overrightarrow{A}(\vec{a})$ or

$\overleftarrow{A}:\overleftarrow{a}\rightarrow[0,1]$, $\overleftarrow{a}\rightarrow\overleftarrow{A}(\overleftarrow{a})$.

We write $(\overleftarrow{A},\overrightarrow{A}) = \overleftarrow{A}$ or $\overrightarrow{A}$, then we named $(\overleftarrow{A},\overrightarrow{A})$ the dynamic fuzzy data sets (DFDS) of U ; you can say that is the membership degree of $(\overleftarrow{A},\overrightarrow{A})$ to the membership function of $(\overleftarrow{A}(\overleftarrow{u}), \overrightarrow{A}(\vec{u}))$. Thus we provided the definition of dynamic fuzzy data sets.

Definition 2. Let(A,A),(B,B)∈DF(U), if $\forall(\overleftarrow{a},\vec{a})\in$(U), $(\overleftarrow{B},\overrightarrow{B})(\overleftarrow{a},\vec{a})\leq(\overleftarrow{A},\overrightarrow{A})(\overleftarrow{a},\vec{a})$ then called $(\overleftarrow{B},\overrightarrow{B})$ is contained in $(\overleftarrow{A},\overrightarrow{A})$ marked as $(\overleftarrow{B},\overrightarrow{B})\subseteq(\overleftarrow{A},\overrightarrow{A})$ and then $(\overleftarrow{A},\overrightarrow{A})=(\overleftarrow{B},\overrightarrow{B})$.Obviously, the relation of contain "$\subseteq$" has features as follows:

(1) Reflexivity:

$\forall$(A,A)∈DF(U), $(\overleftarrow{A},\overrightarrow{A})\subseteq(\overleftarrow{A},\overrightarrow{A})$;

(2) Transitive Characteristic:

$(\overleftarrow{A},\overrightarrow{A})\subseteq(\overleftarrow{B},\overrightarrow{B})$; $(\overleftarrow{B},\overrightarrow{B})\subseteq(\overleftarrow{C},\overrightarrow{C})\Rightarrow(\overleftarrow{A},\overrightarrow{A})\subseteq(\overleftarrow{C},\overrightarrow{C})$

(3) Anti-symmetry:

$(\overleftarrow{A},\overrightarrow{A})\subseteq(\overleftarrow{B},\overrightarrow{B})$; $(\overleftarrow{B},\overrightarrow{B})\subseteq(\overleftarrow{A},\overrightarrow{A})\Rightarrow(\overleftarrow{A},\overrightarrow{A})=(\overleftarrow{B},\overrightarrow{B})$

Definition 3. Let (A,A),(B,B)∈DF(U), then called $(\overleftarrow{A},\overrightarrow{A})\cup(\overleftarrow{B},\overrightarrow{B})$as the union of $(\overleftarrow{A},\overrightarrow{A})$ and $(\overleftarrow{B},\overrightarrow{B})$, and $(\overleftarrow{A},\overrightarrow{A})\cap(\overleftarrow{B},\overrightarrow{B})$ as the intersection of $(\overleftarrow{A},\overrightarrow{A})$ and $(\overleftarrow{B},\overrightarrow{B})$, still we called $(\overleftarrow{A},\overrightarrow{A})^c$ as the complementation of $(\overleftarrow{A},\overrightarrow{A})$. We provided several following membership functions to the three above calculations:

$((\overleftarrow{A},\overrightarrow{A})\cup(\overleftarrow{B},\overrightarrow{B}))(\overleftarrow{a},\vec{a})=(\overleftarrow{A},\overrightarrow{A})(\overleftarrow{a},\vec{a})\vee(\overleftarrow{B},\overrightarrow{B})(\overleftarrow{a},\vec{a})$

$\triangleq\max((\overleftarrow{A},\overrightarrow{A})(\overleftarrow{a},\vec{a}),(\overleftarrow{B},\overrightarrow{B})(\overleftarrow{a},\vec{a}))$;

$((\overleftarrow{A},\overrightarrow{A})\cap(\overleftarrow{B},\overrightarrow{B}))(\overleftarrow{a},\vec{a})=(\overleftarrow{A},\overrightarrow{A})(\overleftarrow{a},\vec{a})\wedge(\overleftarrow{B},\overrightarrow{B})(\overleftarrow{a},\vec{a})$

$\triangleq\min((\overleftarrow{A},\overrightarrow{A})(\overleftarrow{a},\vec{a}),(\overleftarrow{B},\overrightarrow{B})(\overleftarrow{a},\vec{a}))$;

$((\overleftarrow{A},\overrightarrow{A})(a))^c=(1-(\overleftarrow{A},\overrightarrow{A})(\overleftarrow{a},\vec{a}))\triangleq((\overleftarrow{1}-\overleftarrow{A}(\overleftarrow{a}),\overrightarrow{1}-\overrightarrow{A}(\vec{a}))$.

B. Estimate of Dynamic Fuzzy Data

Above, we have talked about the calculation of dynamic fuzzy data. In fact, it is another important content to study estimate of dynamic fuzzy data. Here is an example: "A grows faster than B." In this example, we should measure their growth speed. This is the main content in this section. we can defined the estimate of dynamic fuzzy data as:

Definition 4. Mapping: $\mu:\delta\rightarrow[\overleftarrow{0},\overrightarrow{0}]$, $[\overleftarrow{1},\overrightarrow{1}]$is called DF estimate, if:

(1) $\mu(\overleftarrow{\Phi},\overrightarrow{\Phi})=(\overleftarrow{0},\overrightarrow{0})$, $\mu(\overleftarrow{X},\overrightarrow{X})=(\overleftarrow{1},\overrightarrow{1})$

(2) $(\overleftarrow{A},\overrightarrow{A})\subset(\overleftarrow{B},\overrightarrow{B})\Rightarrow\mu(\overleftarrow{A},\overrightarrow{A})\leq\mu(\overleftarrow{B},\overrightarrow{B})$

(3) $(\overleftarrow{A}_n,\overrightarrow{A}_n)\uparrow(\downarrow)(\overleftarrow{A},\overrightarrow{A})\Rightarrow\mu(\overleftarrow{A}_n,\overrightarrow{A}_n)\uparrow(\downarrow)\mu(\overleftarrow{A},\overrightarrow{A})$

Then $((\overleftarrow{A},\overrightarrow{A}),\ \delta,\ \mu)$ is called the space of DF estimate.

Definition 5. If $g_\lambda\rightarrow[\overleftarrow{0},\overrightarrow{0}]$, $[\overleftarrow{1},\overrightarrow{1}]$ satisfies conditions:

(1) $g_\lambda(\overleftarrow{X},\overrightarrow{X})=(\overleftarrow{1},\overrightarrow{1})$;

(2) $g_\lambda((\overleftarrow{A},\overrightarrow{A})\cup(\overleftarrow{B},\overrightarrow{B}))=$

$g_\lambda(\overleftarrow{A},\overrightarrow{A})+g_\lambda(\overleftarrow{B},\overrightarrow{B})+\lambda g_\lambda(\overleftarrow{A},\overrightarrow{A})g_\lambda(\overleftarrow{B},\overrightarrow{B})$,

$(\overleftarrow{A},\overrightarrow{A})\cap(\overleftarrow{B},\overrightarrow{B})=(\overleftarrow{\Phi},\overrightarrow{\Phi})$

(3) $(\overleftarrow{A}_n,\overrightarrow{A}_n)\uparrow(\downarrow)(\overleftarrow{A},\overrightarrow{A})$

$\Rightarrow\lim_{(\overleftarrow{n},\vec{n})\to\infty}g_\lambda(\overleftarrow{A}_n,\overrightarrow{A}_n)=g_\lambda(\overleftarrow{A}_n,\overrightarrow{A}_n)$

Which is called g_λ estimate.

Theorem 1. While $(\overleftarrow{\lambda},\overrightarrow{\lambda})>(-\overleftarrow{1},-\overrightarrow{1})$ then we call g_λ estimate DF estimate.

Theorem 2. $g_\lambda(\overleftarrow{\lambda},\overrightarrow{\lambda})>(-\overleftarrow{1},-\overrightarrow{1})$ has features as follows:

(1) $g_\lambda(\overleftarrow{\lambda},\overrightarrow{\lambda})=\dfrac{(\overleftarrow{1},\overrightarrow{1})-g_\lambda(\overleftarrow{A},\overrightarrow{A})}{(\overleftarrow{1},\overrightarrow{1})+\lambda g_\lambda(\overleftarrow{A},\overrightarrow{A})}$

(2) $g_\lambda(\overleftarrow{A},\overrightarrow{A})\cap(\overleftarrow{B},\overrightarrow{B})$

$=\dfrac{g_\lambda(\overleftarrow{A},\overrightarrow{A})+g_\lambda(\overleftarrow{B},\overrightarrow{B})-g_\lambda(\overleftarrow{A},\overrightarrow{A})(\overleftarrow{B},\overrightarrow{B})+\lambda g_\lambda(\overleftarrow{A},\overrightarrow{A})g_\lambda(\overleftarrow{B},\overrightarrow{B})}{(\overleftarrow{1},\overrightarrow{1})+\lambda g_\lambda(\overleftarrow{A},\overrightarrow{A})(\overleftarrow{B},\overrightarrow{B})}$

C. DFL Proposition Calculus

Definition 6. A statement having character of dynamic fuzzy is called dynamic fuzzy proposition that is usually symbolized by capital letter A , B, C⋯

Definition 7. A dynamic fuzzy number $(\overleftarrow{a},\vec{a})\in[0,1]$ which is used to measure a dynamic fuzzy proposition's true or false degree is called dynamic fuzzy proposition's true or false. It is usually symbolized by$(\overleftarrow{a},\vec{a})$, $(\overleftarrow{b},\vec{b})$, $(\overleftarrow{c},\vec{c})$, Here $(\overleftarrow{a},\vec{a})=\overleftarrow{a}$ or $\vec{a}$, $\min(\overleftarrow{a},\vec{a})\triangleq\overleftarrow{a}$, $\max(\overleftarrow{a},\vec{a})\triangleq\vec{a}$, the same are as follows.

Definition 8. A dynamic fuzzy propositions can be regarded as a variable whose value is in the interval [0,1]. The variable is called dynamic fuzzy proposition variable that is usually symbolized by small letter.

Definition 9. Dynamic fuzzy calculus formation can be defined as follows:

(1) A simple dynamic fuzzy variable itself is a well formed formula.

(2) If $(\overleftarrow{x},\vec{x})$p is a well formed formula, then $\overline{(\overleftarrow{x},\vec{x})p}$is a well formed formula, too.

(3) If ($\overleftarrow{x}, \overrightarrow{x}$)P and ($\overleftarrow{y}, \overrightarrow{y}$)Q are well formed fomulas, then ($\overleftarrow{x}, \overrightarrow{x}$)P $\vee$ ($\overleftarrow{y}, \overrightarrow{y}$)Q, ($\overleftarrow{x}, \overrightarrow{x}$)P $\wedge$ ($\overleftarrow{y}, \overrightarrow{y}$)Q, ($\overleftarrow{x}, \overrightarrow{x}$)P$\rightarrow$ ($\overleftarrow{y}, \overrightarrow{y}$)Q, ($\overleftarrow{x}, \overrightarrow{x}$)P $\leftrightarrow$ ($\overleftarrow{y}, \overrightarrow{y}$)Q are also well formed formulas.

(4) A sting of symbols including proposition variable connective and brackets is well formed formula if and only if the strings can be obtained in a finite number of steps, each of which only applies the earlier rules(1), (2) and (3).

D. DFL Predicate Calculus

Definition 10. Recursive definition of DFL predicate is as follows:

(1) An atom (first order logical symbols) is a formula.

(2) If G and H are formulas, T is a dynamic fuzzy truth value of assignment, ($\overleftarrow{x}, \overrightarrow{x}$) is a free variable in DFL , then G$\vee$H, G$\wedge$H, G$\rightarrow$H, G $\leftrightarrow$ H, $\overline{G}$, ($\forall$($\overleftarrow{x}, \overrightarrow{x}$)G), ($\exists$($\overleftarrow{x}, \overrightarrow{x}$)G) are all formulas.

(3) Any string of symbol is a formula of DFL, if and only if the string can be obtained in a finite of steps, each of which only applies the earlier rules (1) and (2).

Definition 11. An explanation of a DFL formula G consists of a non-empty domain and some rules as follows:

For every variable symbol in G, a dynamic fuzzy element is assigned,

For every n-array function symbol in G, a mapping U$\xrightarrow{T}$D is assigned,

For every n-array predicate symbol in G, a mapping D$\xrightarrow{T}$B is assigned.

where B is a dynamic fuzzy logic predicate system.

IV. CURVELET FUSION BY ENTROPY AND DFL

The goal of using curvelets for image fusion is to synthesize a new image, which can provide more abundant information than any one of the individual original images.

In this paper, a new model which is a quintuple in order to assess the fusion degree for DSA medical image optimum adjustment by DFL is proposed to fusion process.

FM=(P, N, E, W, S)

Where P is a series of DSA medical images that needs to be fused, P={P_1,..., P_{n-1}, P_n};N is the number of levels of curvelet transform within P; E is the entropy of the certain levels of image after curvelet transformation; W is a set of the weight of each factor of N and E; Q is quality scores set of all the DSA images within P, Q={Q_1, Q_2,…, Q_n}. The final fusion adjustment degree for image subbands coefficients is decided by the order of Q .

The model has some attributes as follows:

- The number of levels of curvelet transform could be calculated by the entropy from different level, the large number means high quality but cost more time.
- The entropy of the certain levels of image after curvelet transformation is a objective parameter of subbands coefficients compution.
- We adjust the value of W for the purpose of computer the quality scores of entropy and levels of curvelet transform levels .Because the value of W is in the interval [0,1], so we could use DFL to construct a membership function to adjust the weight for DSA image fusion.

We define the domain of discouse ($\overleftarrow{U}, \overrightarrow{U}$)=[$\overleftarrow{0}, \overrightarrow{0}$], [$\overleftarrow{1}, \overrightarrow{1}$]=[0,1]$\times$[$\leftarrow$,$\rightarrow$], dynamic fuzzy data sets ($\overleftarrow{A}, \overrightarrow{A}$) and ($\overleftarrow{B}$, $\overrightarrow{B}$) indicate “enhancement” and “reduction” respectively. Where “ $\leftarrow$ ” denotes increasing or advance direction of dynamic change and “ $\rightarrow$ ” indicates reducing or back direction of dynamic change.

For the member of DFD $W \in$[0,1], then W belongs to DF sets defined as: $W \stackrel{DF}{=} (\overleftarrow{w}, \overrightarrow{w}) = \overleftarrow{w}$ or $\overrightarrow{w}$ and $\max(\overleftarrow{w}, \overrightarrow{w}) \stackrel{\Delta}{=} \overrightarrow{w}$, $\min(\overleftarrow{w}, \overrightarrow{w}) \stackrel{\Delta}{=} \overleftarrow{w}$

The calculation between two DFD subsets can be comprehended absolutely as the calculation between its membership function. We construct the membership function as follows:

$$\overleftarrow{A}(\overleftarrow{u}) = \begin{cases} 0 & \text{if } 0 \le \overleftarrow{u} \le \overleftarrow{a} \le \overleftarrow{1} \\ (1+(\dfrac{\overleftarrow{u}-a}{0.1\times a})^{-2})^{-1} & \text{if } 0 \le \overleftarrow{a} \le \overleftarrow{u} \le \overleftarrow{1} \end{cases}$$

$$\overrightarrow{A}(\overrightarrow{u}) = \begin{cases} 0 & \text{if } 0 \le \overrightarrow{u} \le \overrightarrow{a} \le \overrightarrow{1} \\ (1+(\dfrac{\overrightarrow{u}-a}{0.1\times a})^{-2})^{-1} & \text{if } 0 \le \overrightarrow{a} \le \overrightarrow{u} \le \overrightarrow{1} \end{cases}$$

$$\overleftarrow{B}(\overleftarrow{u}) = \begin{cases} 0 & \text{if } 0 \le \overleftarrow{u} \le \overleftarrow{3a} \le \overleftarrow{1} \\ (1+(\dfrac{\overleftarrow{u}-3a}{0.1\times 3a})^{-2})^{-1} & \text{if } 0 \le \overleftarrow{3a} \le \overleftarrow{u} \le \overleftarrow{1} \end{cases}$$

$$\overrightarrow{B}(\overrightarrow{u}) = \begin{cases} 0 & \text{if } 0 \le \overrightarrow{u} \le \overrightarrow{3a} \le \overrightarrow{1} \\ (1+(\dfrac{\overrightarrow{u}-3a}{0.1\times 3a})^{-2})^{-1} & \text{if } 0 \le \overrightarrow{3a} \le \overrightarrow{u} \le \overrightarrow{1} \end{cases} \quad (20)$$

By using the constructed membership function of DFL, we could find the trend of fusion quality, so the performance of fusion based on DFL is prominent.

V. EXPERIMENTAL RESULTS AND DISCUSSION

Before the experiment, we get a lot of CT images from the First Affiliated Hospital of Soochow University, including the brain’s images and DSA images. All images are follow DICOM standard.

A. The Experimental using DFL

In order to fusion the DSA medical serial images, we proposed a new fusion method combine curvelet transform with DFL. The images of the DSA medical serial images focus on the same scene at different time are shown in Figure 3.

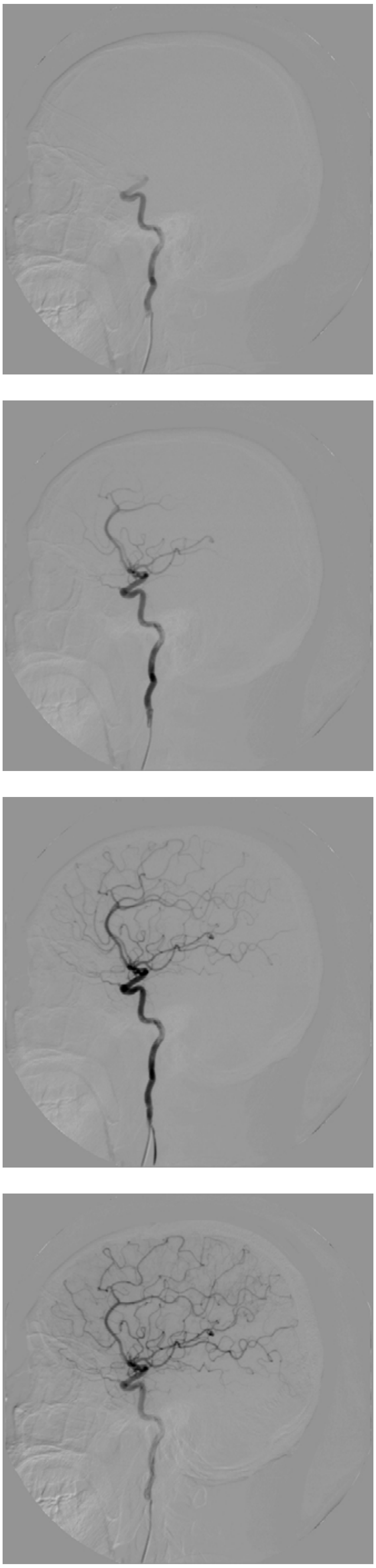


Figure 3. DSA Medical Serial Image

In our experiment, we fused several images into one. The steps of this processing method are as follows:

- Each of DSA serial images should be decomposed into different subbands based on digital curvelet transform.
- Construct a membership function based on DFL to adjust the subbands coefficients via entropy.
- Curvelet reconstruction from different subband to synthesize one image which contain the details of blood vessels from each single DSA image.

The steps we presented are illustrated as follows:

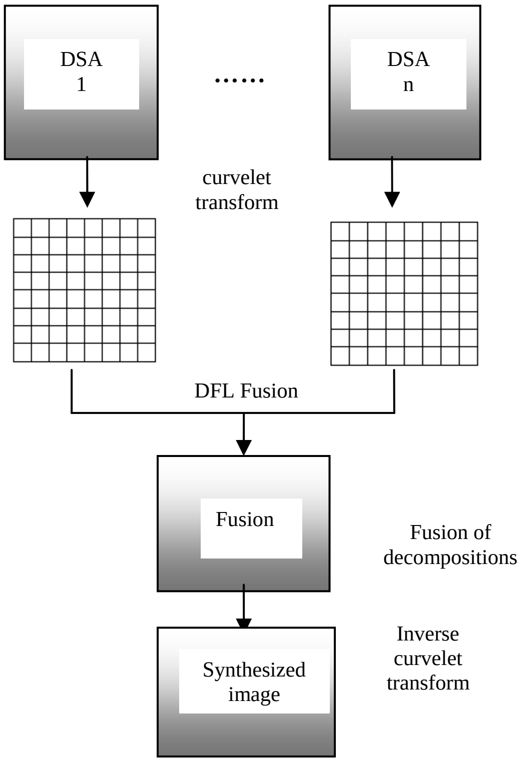


Figure 4. Image Fusion based on Curvelet Transform and DFL

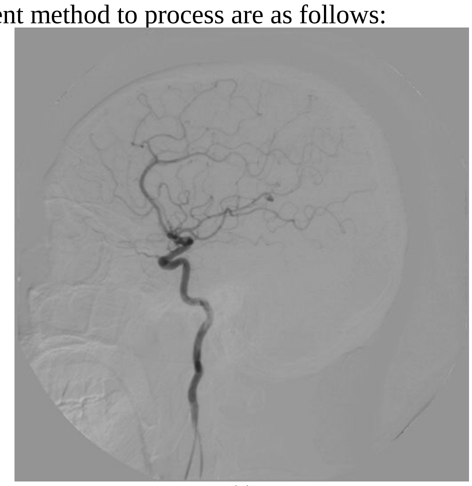
At the beginning we could set the image size as 256 ×256. It should be processed of size n by n, such as n = $2^8 = 4^4$. By this setting we would easily to extend the construction of transforms for n = 1024= 2^{10} and n = 4096= 2^{12} or so as to fit other sizes of medical image.

Via curvelet transform of a DSA image as size of 256×256, we could obtain 3 arrays as the subbands s = 1, 2, and 3 respectively. There are $(2^4)^2$ =256 coefficients contained in the subband. Each slice which output for every Subbands could be viewed as an image. There are a lot of small squares has been embedded in this slice of image.

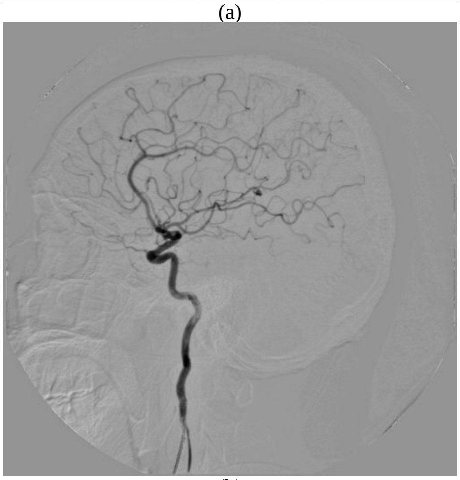
At last the DSA image fusion method based on DFL and curvelet entropy could be applied in each slice respectively from different subband to synthesize one reconstruction medical image for computer-aided diagnostics.

B. Results and Discussion

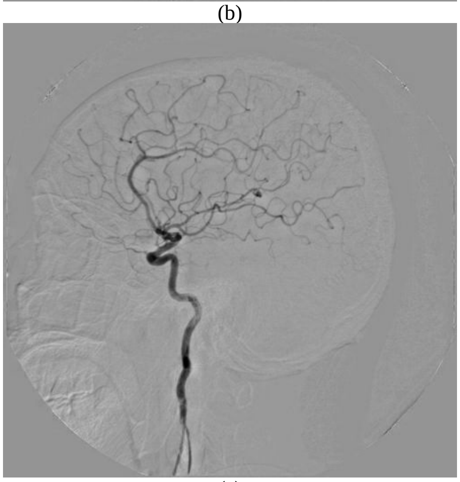
In order to testify this paper's method is superior, we use other method such as weighted average method, laplacian pyramid method and traditional curvelet transform method to process the source DSA serial images fusion. By contrast, the efficiency of DFL curvelet transform fusion approach which is shown in Figure 5(d) is better than weighted average method, laplacian pyramid method and traditional wavelet transform method which is shown in Figure 5(a), (b) and (c) respectively. Many details were reserved in Figure 5(d). The synthesized image of blood vessels looks much more clearly than that in the other images. The results of different method to process are as follows:



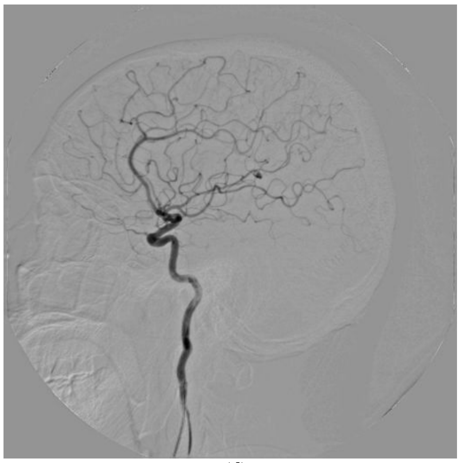
(a)



(b)



(c)



(d)

Figure 5. Contrast Several Method of DSA Medical Image Fusion

The quality of image fusion should be evaluated objectively. Image entropy reflects the number of average information which the image contains is often used to test the effect. The formula can be expressed as follows:

$$\mathrm{E} = \sum_{\mathrm{k=0}}^{\mathrm{L-1}} P_k \log(P_k) \tag{3}$$

Where E is the entropy of image, P_k is the probability that gray value equivalent to k, that is the radio of the number of pixels whose value is k to the number of pixels in total. L is the amount for grey value of the image.

A standard objective measure of image quality in this paper is reconstruction error. There are two of the error metrics used to compare the different image fusion techniques. They are root mean square error (RMSE) and the signal to noise ratio (SNR). The RMSE is the cumulative squared error between the merged and the original image, and SNR is a measure of the image error. The mathematical formulae can be expressed as follows:

$$RMES = \sqrt{\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(I_{ij} - I_{ij}')^2} \tag{4}$$

$$\mathrm{SNR} = 10\lg[\sum_{\mathrm{i=1}}^{\mathrm{M}}\sum_{\mathrm{j=1}}^{\mathrm{N}}(I - I'')^2 / \sum_{\mathrm{i=1}}^{\mathrm{M}}\sum_{\mathrm{j=1}}^{\mathrm{N}}(I - I')^2] \tag{5}$$

In these formulae, I(x, y) is the reference image, I'(x, y) is the fusion image, I'' (x, y) is the mean value and M, N are the dimensions of the images. A lower RMSE means the fusion image has lesser error. And a higher SNR means the effect of image fusion is good because it shows that the signal is more to the noise in this image.

TABLE I.
EVALUATION OF VARIOUS FUSION METHODS

Processing Method	Entropy	RMSE	SNR
weighted average	5.083	4.857	35.336
laplacian pyramid	5.110	4.403	35.734
traditional DWT	5.122	4.107	36.086
our method	5.223	3.925	37.528

From the analysis of the results of various methods, it is inferred that the effect of DSA image fusion method based on dynamic fuzzy logic and curvelet entropy have a higher value of entropy and SNR, a lower value of RMSE. It indicates our method is superior to weighted average, laplacian pyramid and traditional DWT methods.

VI. CONCLUSIONS

In this paper, the characters of DSA medical serial images were analyzed for image fusion. We have developed a novel method which combine curvelet transform with dynamic fuzzy logic theory. Firstly, we decompose the image by curvelet transform, secondly we use an improved fusion approach based on DFL in each level to adjust the weight for image subbands coefficients via entropy. At last, we reconstruct the image by inverse curvelet transform to synthesize one DSA medical image which could contain more integrated accurate detail information of blood vessels than any one of the individual source images. Further investigations on the use of dynamic fuzzy logic and curvelet transform to 3-D DSA medical image processing are left for future work.

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